GLOBAL EDITION

Mathematical Reasoning

for Elementary Teachers

SEVENTH EDITION

Calvin T. Long • Duane W. DeTemple • Richard S. Millman

ALWAYS LEARNING

PEARSON

Mathematical Reasoning

FOR

ELEMENTARY TEACHERS

This book presents the mathematical knowledge needed for teaching, with an emphasis on why future teachers are learning the content, as well as when and how they will use it in the classroom. The Seventh Edition teaches the ***content in context*** to prepare today's students for tomorrow's classroom.

The **Common Core State Standards for Mathematics** include **8 Standards for Mathematical Practice (SMP)**, which have been integrated throughout this text. It's important for future teachers to know what will be expected of them when they are in the classroom, and these SMP references ensure that future teachers be both familiar and comfortable with these mathematical practices. Instances where an SMP applies are called out with an icon and highlighted text.

Continuing from the Common Core, the following eight Standards for Mathematical Practice are designed to teach students to

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice elaborate and reinforce the importance of Pólya's four principles of problems solving. In particular, special attention is given to the fourth principle, to

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quantities and their operation including differing units, such as cm, cm^2, cm^3, Fahrenheit versus Celsius temperature, and so on. Computations with different units can cause a real change in a problem. Unfortunately, you will see an example of a disaster in the paragraph immediately after SMP 2.

"Mathematically proficient students make sense of quantities and their relationships in problem situations . . . Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them. . . ."

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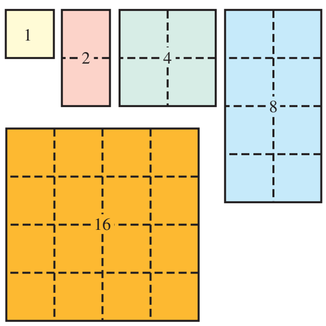
COOPERATIVE INVESTIGATION

Numbers from Rectangles

Material Needed

1. One rectangle of each of these shapes for each student:



2. One record sheet like this for each student:

	16	8	4	2	1
0	0	0	0	0	0
1	0	0	0	0	1
18	1	0	0	1	0
19	1	0	0	1	1

	16	8	4	2	1
20	1	0	1	0	0
21					
38					
39					

Directions

Step 1. Use the rectangles to determine whether or not there are representations of each of the numbers 0, 1, 2, . . . , 39 as a sum of the numbers 1, 2, 4, 8, or 16, with each of the latter group of numbers used at most once.

Step 2. For each representation determined in step 1, record the numbers (rectangles) used by placing a 0 or a 1 in the appropriate columns of the record sheet. The rows for 0, 1, 18, 19, and 20 have been done for you.

(a) Do all the numbers from 0 through 39 have such a representation?

(b) What additional numbers could be represented if you had a 32 rectangle?

(c) Describe any interesting patterns you see on your record sheet.

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COOPERATIVE INVESTIGATIONS begin each chapter, offering content-related games and puzzles that motivate the chapter. These can be easily adapted for use in the elementary classroom.

INTO THE CLASSROOM

Ann Hlabangana-Clay Discusses the Addition of Fractions

I use red shoelace licorice to introduce adding fractions. It is flexible and tangible for small fingers to demonstrate whole to part. To start the lesson, I give each student one whole red shoelace licorice. I ask them to spread it out from end to end and use it to measure a starting line and an ending line. To find $\frac{1}{2} + \frac{3}{4}$, I give each student a $\frac{1}{2}$ length shoelace and have them measure it against the whole. Each student also gets a $\frac{3}{4}$ length shoelace to measure against the whole and to compare to the $\frac{1}{2}$. After comparing the $\frac{1}{2}$ and the $\frac{3}{4}$, I have the students connect the two shoelaces together and share their findings with their partner.

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Our completely updated **INTO THE CLASSROOM** feature provides insights from real elementary and middle school teachers related to various topics throughout the text, as students in this course are thinking ahead to when they will have classrooms of their own. Along with the feature, we have added new Into the Classroom problems into the problem sets. These problems pose questions that will help future teachers consider how they might clarify subtle and often misunderstood points for their future students.

Into the Classroom

15. (**Writing**)

(**a**) How would you use colored counters to help students understand that $-(-4) = 4$?

(**b**) How would you use colored counters to help students understand that $-(-n) = n$ for every integer n (positive, negative, or 0)?

16. (**Writing**) The definition of absolute value is often confusing to students. On the one hand, they understand that the absolute value of a number is always positive. On the other hand, the definition states that $|n| = -n$ sometimes. How would you explain this seeming contradiction?

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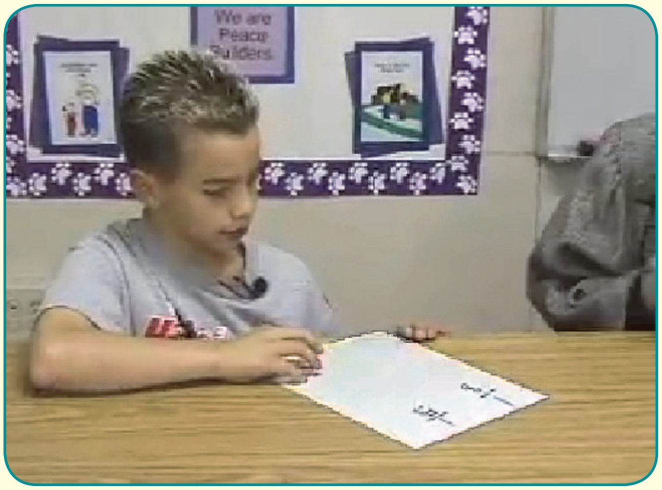

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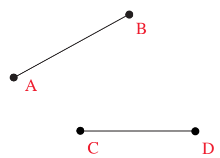
Integrating Mathematics and Pedagogy (IMAP) videos, available in MyMathLab, enable future teachers to see elementary and middle school students working out problems and explaining their thought processes. IMAP videos often help future students understand why they need to understand the elementary and middle school mathematics at a deeper conceptual level.

Responding to Students exercises give insight into the mathematical questions and procedures that children will come up with on their own, and offer ways to respond to them.

Responding to Students

29. Larisa, a second grader, is asked whether the following segments drawn on a page are parallel:



After a brief pause, she says, "Yes, they are because they don't meet." How would you respond to Larisa?

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7TH EDITION

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Dedication

To the memory of my good wife
and constant helpmate, Jean. ▶ **C.T.L.**

To my wife, Janet, and my daughters,
Jill and Rachel. ▶ **D.W.D.**

To Sandy, for her loving
support. ▶ **R.S.M.**

Contents

*Available in MyMathLab

Preface

TO THE FUTURE TEACHER

You may be wondering what to expect from a college course in mathematics for prospective elementary or middle school teachers. Will this course simply repeat arithmetic and other material that you already know, or will the concepts be new and interesting? In this preface, we will give a positive answer to that question and at the same time provide a useful orientation to the text.

This book is designed to help you, as a future teacher, add to the depth of your knowledge about the mathematics of elementary and middle school. Most institutions structure their teacher education curriculum to start with a sequence of mathematics content courses. The content courses serve as a prerequisite for the teaching methods course, which deals with, among other ideas, how school children learn mathematics as they grow and develop. This text is filled with activities, investigations, and a host of problems with results and answers that are attractive, surprising, and unexpected, yet are designed to engage you thoughtfully doing mathematics.

The content needed for future teachers is covered fully and done so with an eye toward giving you a deep background into why things work (and why some things don't). Emphasis is placed on **the mathematical knowledge needed for teaching**, a topic very much a part of research in mathematics education today. This depth, called **conceptual understanding,** is a very important part of being a teacher. To decide whether the methods or ideas of your students are right or wrong and be able to explain to the students why is one of the most important aspects of teaching. In addition, the depth of your confidence and basic skills will be increased during this course as you participate in solving problems and performing operations in a number of different ways.

The **Common Core State Standards for Mathematics** (CCSS-M), or **Common Core**, is a new approach to the curriculum in mathematics and has been widely adopted in the United States. It has influenced this text significantly as you will soon see. In addition to content, the CCSS-M also advocates *pedagogy* (the practice of teaching). As of 2013, 46 states plus the District of Columbia are now working with the **Common Core** in mathematics.

Problem Solving and Mathematical Reasoning

Problem solving (or, said another way, "mathematical reasoning") is stressed throughout as a major theme of this book.

At first, problem solving may seem daunting, but don't be afraid to try and perhaps not succeed, because you will succeed as you keep trying. As you gain experience and begin to acquire an arsenal of strategies, you will become increasingly successful and will even begin to find the challenge of solving a problem stimulating and enjoyable. Quite often, with much surprise, this has been the experience of students in our classes as they successfully match wits with problems and gain insights and confidence that together lead to even more success.

You should not expect to see instantly into the heart of a problem or to immediately know how it can be solved. The text contains many problems that check your understanding of basic concepts and build basic skills, but you will also continually encounter problems requiring multiple steps and reflection. These problems are not unreasonably hard. (Indeed, many would be suitable for use in classes you will subsequently teach with only minor modifications.) However, they do require thought. Expect to try a variety of approaches, be willing to discuss possibilities with your classmates, and form a study group to engage in cooperative problem solving. This is the way mathematics is done, even by professionals, and as you gain experience, you will increasingly feel the real pleasure of success and the beginning development of a mathematical habit of your mind.

You will greatly improve your thinking and problem-solving skills if you take the time to write carefully worded solutions that explain your method and reasoning. Similarly, it will help you engage in mathematical conversations with your instructor and with other students. Research shows that mechanical skills learned by rote without understanding are soon forgotten and guarantee failure, both for you now and for your students later. By contrast, the ability to think creatively makes it more likely that the task can be successfully completed. *Conceptual understanding of the material is the key to your success and the future success of your students.*

How to Read This Book

Learning mathematics is not a spectator sport.

No mathematics textbook can be read passively. To understand the concepts and definitions and to benefit from examples, you must be an active participant in a conversation with the text. Often, this means that you need to check a calculation, make a drawing, take a measurement, construct a model, or use a calculator or computer. If you first attempt to answer questions raised in the examples on your own, the solutions written in the text will be more meaningful and useful than they would be without your personal involvement.

The odd problems are fully or partially answered in the back of the book. These answers give you an additional source of worked examples. But again, you will benefit most fully by attempting to solve the problems on your own or in a study group before you check your reasoning by looking up the answer provided in the text.

Guiding Philosophy and Approach

The content and processes of mathematics are presented in an appealing and logically sound way with these major goals in mind:

- to develop positive attitudes toward mathematics and the teaching of mathematics,
- to develop mathematical knowledge and skills, with particular emphasis on problem solving and mathematical reasoning,
- to develop a conceptual understanding of the mathematics of elementary and middle school,
- to develop excellent teachers of mathematics, and
- to understand the three major themes below of the mathematics and the ability to teach.

Three Major Themes

This text responds to three overarching themes that shape the content (conceptual development) and pedagogical skills (teaching excellence) required for the successful elementary or middle school teacher. The first of these themes is recognition of the *Principles and Standards for School Mathematics,* first set forth in 1989 by the National Council of Teachers of Mathematics (NCTM) and revised in 2000 to its current form. The second theme is problem solving, exemplified by Principles of Problem Solving, set forth in George Pólya's classic book *How to Solve It,* first published in 1945 and rewritten beautifully in 1988. Finally, the third and most recent theme is the recognition of the content and teaching standards found the *Common Core State Standards—Mathematics.*

The *Principles and Standards* of the NCTM, *Pólya's Problem Solving Principles,* and the *Common Core State Standards* at first may look different, but they really are not. They share a common vision of teaching skills and conceptual understandings of effective teachers of mathematics and they provide different ways to look at mathematical reasoning.

You will find the three themes infused in this text in the following ways:

- *The Principles and Standards of the NCTM*

  The goals of NCTM *Principles and Standards* for school mathematics are to "ensure quality, indicate goals, and promote positive changes in mathematics education in grades preK-12."

  An overview of the *Principles and Standards* is contained at the back of the book and should be reviewed at different times during the semesters as you work to look forward to "big picture" ideas.

- *Pólya's Principles of Problem Solving*

  The four principles of Pólya will be of tremendous help to you throughout this text. They will be used frequently in the early chapters, in particular, and are listed as

  Pólya's First Principle: Understand the Problem
  Pólya's Second Principle: Devise a Plan
  Pólya's Third Principle: Carry Out the Plan
  Pólya's Fourth Principle: Look Back

  Special attention is given to the fourth principle, "Look Back."
- *The Common Core State Standards—Mathematics*

  The mathematics topics included in this text thoroughly cover the content standards set forth in the CCSS-M. In addition to the content standards, the CCSS-M sets forth eight **Standards for Mathematical Practice**. (A complete statement of the standards can be found at the end of this book.) The goal of these standards is to ensure that teachers instill the following skills and approaches to reasoning in their students:

  1. Make sense of problems and persevere in solving them.
  2. Reason abstractly and quantitatively.
  3. Construct viable arguments and critique the reasoning of others.
  4. Model with mathematics.
  5. Use appropriate tools strategically.
  6. Attend to precision.
  7. Look for and make use of structure.
  8. Look for and express regularity in repeated reasoning.

We encourage you to read and compare throughout the ideas of the NCTM *Principles and Standards,* Pólya's Principles, and the *Common Core State Standards for Mathematics*. Excerpts and examples illustrating the standards and problem solving strategies are provided in every chapter. For example, when we are working an example, describing a concept, examining a definition, or solving an insightful problem, there will be times when you (and sometimes your students) will work in-depth to solve problems. Notions like exploring, explaining or expanding blend ideas from these three parts of the text to help readers recognize the mathematics that increases their mathematical habits of the mind.

Mathematical habits of mind are studied in mathematics education and have been used in earlier editions of this text. Since the mathematical habits of mind are very much a part of NCTM *Principles and Standards,* Pólya's Principles, and the *Common Core State Standards for Mathematics,* we will not continue to formally use that notion in this edition.

This text models effective teaching by emphasizing:

- manipulatives
- investigations
- activities for discovery
- written projects
- discussion questions
- appropriate use of technology

And, above all else,

- **problem solving**, **mathematical reasoning**, and **conceptual understanding.**

New to This Edition

The Common Core State Standards—Mathematics. The eight goals of the **Standards for Mathematical Practice** (SMP) are designed to teach our students to combine the mathematical practice (to see what is happening!) and to understand the mathematical content. The eight SMP principles describe ways in which future teachers "increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise." The SMP also places importance on the five NCTM process standards, which are found in the back of this text together with the five content standards. An important new idea is to make sure that the SMP can be "seen" in the combination of teaching and content. We have provided examples of the SMP goals in this text giving a way for you, as future teachers, to see what will happen when you are teaching.

Different Bases. Chapter 3 is reorganized in this edition in response to suggestions by instructors who felt that arithmetic in bases other than ten needed to be grouped completely in one section. To follow that approach, the authors have included all of base ten and its arithmetic (addition, subtraction, multiplication, and division) into the first three sections. The fifth section contains the non-decimal positional system with an emphasis on bases of five or six. However, while addition, subtraction and multiplication will be done, division in bases other than ten is not covered in the text as it increases significantly both the length and the complexity of long or short division. Some instructors may want to omit bases other than ten, which is certainly appropriate, and so Section 3.5 is optional and may be skipped. The other section, Section 3.4, is important, however, as it emphasizes estimation and mental arithmetic for elementary students (in base ten, of course.)

Into the Classroom. Completely new "Into the Classroom" features provide insights from active teachers, providing a window into their classroom via activities, projects, discussions, and ideas to engage children in the mathematics being covered in that particular chapter of the text. These features help students make connections between what they are currently learning in this textbook and what they will be teaching in their future classrooms.

Integrating Mathematics and Pedagogy (IMAP). IMAP video references allow future teachers to see elementary and middle school students working out numerical concepts. These videos provide an opportunity for valuable classroom discussion of the mathematics and knowledge of student understanding needed to teach concepts. The IMAP videos are available in MyMathLab.

Responding to Students Problems (RTS). Throughout the problem sets there are many new problems that give examples of the ways in which children try to use mathematical techniques. A really important part of being an excellent teacher is to be able to analyze what the children are doing and then give them help at a conceptual level or show them why their method works. The RTS problems show that future teachers will need a thorough understanding of mathematical content in order to answer students' questions. We want to thank Jean Anderson, who has 25 years of experience teaching in elementary and middle school in DeKalb, Georgia, for her contributions to the RTS problems and to Cameron Schriner for his help in constructing figures and tables in some problem sets.

Probability. The chapter has been completely revised to provide a comprehensive introduction to elementary probability for future K–8 teachers. With this new approach, the basics of probability are introduced quickly by their placement in the opening section. The next two sections develop counting principles with immediate applications to the calculation of probabilities; in this way, the importance of counting principles is readily apparent. The final section takes up selected additional topics—odds, expected values, geometric probability, and simulations, so that the chapter as a whole provides the background required of future teachers to meet the content standards of the NCTM and the **Common Core**.

Overview of Content

- **Problem Solving** We begin the text with an extensive introduction to problem solving in Chapter 1. This theme continues throughout the text in special problem-solving examples and is featured in the problems grouped under the headings "Thinking Critically," "Into the Classroom," and "Making Connections." New in this seventh edition is the use of the recently added Section 1.4, "Algebra as a Problem-Solving Strategy," as a platform for the expanded Chapter 8, which applies algebra to geometry.
- **Number Systems** Chapters 2, 3, 5, 6, and 7 focus on the various number systems and make use of discussion, pictorial and graphical representations, and manipulatives to promote an understanding of the systems, their properties, and the various modes of computation. There is plenty of opportunity for drill and practice, as well as for individual and cooperative problem solving, reasoning, and communication.
- **Number Theory** Chapter 4 contains much material that is new, interesting, and relevant to students' careers as future teachers. Notions of divisibility, divisors, multiples, greatest common divisors, greatest common factors, and least common multiples are developed first via informative diagrams and then through the use of manipulatives, sets, prime-factor representations, and the Euclidean algorithm.

- **Algebraic Reasoning and Representation** Although algebraic notions are used earlier in the text, Chapter 8 gives a careful and readable discussion of algebraic ideas needed in elementary and middle school. Included in the discussion are variables; algebraic expressions and equations; linear, quadratic, and exponential functions; simple graphing in the Cartesian plane; and especially the intimate relationship between algebra and geometry, in the last section of the chapter. All of these concepts are increasingly appearing in texts for elementary and middle school students. Schoolteachers must therefore understand algebraic and geometric ideas to be comfortable teaching from current texts. Chapter 8, however, is not meant as a comprehensive review of algebra. Its focus is on the algebra that is a part of the elementary and middle school curriculum.
- **Geometry** The creative and intuitive nature of geometric discovery is emphasized in Chapters 9, 10, 11, and 12. These chapters will help students view geometry in an exciting new way that is much less formal than they have seen before. The text's approach to geometry is constructive and visual. Students are often asked to draw, cut, fold, paste, count, and so on, making geometry an experimental science.

  Problem solving and applications permeate the geometry chapters, and sections on tiling and symmetry provide an opportunity to highlight the aesthetic and artistic aspects of geometry. Examples are taken from culturally diverse sources. Though it is optional, many of the concepts and construction of geometry are enhanced by their exploration with dynamic geometry software such as GeoGebra, Geometer's Sketchpad, and the like.
- **Statistics** Chapter 13, on statistics, is designed to give students an appreciation of the basic measures and graphical representations of data. The examples and problem sets use updated data and are also relevant for the children and the future teachers as the problems and examples are focused on education. This section has been modified in this edition to include "Responding to Students" problems and many new State Assessment problems. To show that statistics really is a part of the elementary school curriculum, 18 State Assessment problems are now included in Sections 13.1 and 13.2. There is also a discussion of the standardized normal distribution, as well as of z scores and percentiles.
- **Probability** Chapter 14 has been completely rewritten, with the first section introducing both *experimental probability*—probability based on experiences and repeated trials—as well as *theoretical probability*—that based on counting and other *a priori* considerations. Many of the fundamental terms and notations are covered in this introductory section, including outcome of a trial, sample space, event, and probability function. Abundant examples are given to clarify the concepts of equally likely outcomes, mutually exclusive events, complementary events, and independent events. The following two sections introduce the principles of counting—the addition and multiplication principles, combinations and permutations. These demonstrate their importance in the determination of theoretical probabilities. The concluding section completes the chapter's introduction to basic probability by discussing odds, expected values, geometric probability, and simulations.

Topics of Special Interest

The text includes several topics that many students will find especially interesting. These topics provide stimulating opportunities to hone such mathematical reasoning skills as problem solving, pattern recognition, algebraic representation, and use of calculators. The following topics are threaded into several chapters and problem sets;

- **The Fibonacci Number and the Golden Ratio.** The Fibonacci numbers (1, 1, 2, 3, 5, 8, . . .) and the Golden Ratio have surprised and fascinated people over the ages and continue to serve as an unlimited source for mathematical and pedagogical examples. It is not always obvious that there is a connection to the Fibonacci numbers. Much of the charm of such exercises consists in the surprise of discovery in unexpected places.
- **Pascal's Triangle.** This well-known triangular pattern that has roots in ancient China has unexpected applications to counting the number of paths through a square lattice and is replete with patterns awaiting discovery.
- **Triangular Numbers.** The numbers in the third column of Pascal's triangle (1, 3, 6, 10, 15, 21, . . .) appear in almost countless unexpected contexts.
- **Magic Squares and Other Magic Patterns.** These topics provide interesting practice in basic number patterns and number facts.

Features for the Future Classroom

A teacher of mathematics should be aware of both the current and historical development of mathematics, have some knowledge of the principal contributors to mathematics, and realize that mathematics continues to be a lively area of research. The text contains a number of features that future teachers will find to be valuable in the classroom:

- **Pólya Principles** have been used in an increasing number of examples, with solutions written to highlight his four-step approach to problem solving—an approach that will be quite useful.
- New! **Common Core State Standards—Standards of Mathematical Practice**: It's important for future teachers to have a comfort level with what will be expected of them when they are in the classroom. The authors provide opportunities in context for you to become more familiar with the Standards of Mathematical Practice and how they relate to the content.
- **NCTM Principles and Standards for School Mathematics**: Classroom teachers appreciate the guidance offered by this document of continuing importance. The six principles address equity, curriculum, teaching, learning, assessment, and technology. The five content standards cover number and operations, algebra, geometry, measurement, and data and probability. The five process standards speak to problem solving, reasoning and proof, communications, connections, and representation.
- New & Improved! **Into the Classroom** provides insights from active teachers, providing a window into their classroom via activities, projects, discussions, and ideas to engage children in the mathematics being covered in that particular chapter of the text. These features help students make connections between what they are currently learning in this textbook and what they will be teaching in their future classrooms.
- New & Improved! **Cooperative Investigations** are activities within the body of the chapters that use small groups to explore the concepts under discussion. Working together is an important skill for future teachers as well as their future students. Additional activities can be found in MyMathLab and the corresponding. Activities Manual by Dolan et al.
- **Integrating Mathematics and Pedagogy (IMAP) videos** provide an opportunity to see children solve real problems and explain their problem solving process. These videos provide a glimpse of what a future classroom may be like and reinforce why a deeper conceptual understanding of mathematics is important for teachers.
- **Highlights from History** illustrate the contributions individuals have made to mathematics and provide a cultural, historical, and personal perspective on the development of mathematical concepts and thought.

Chapter Elements

The chapters following Chapter 1 are consistently and meaningfully structured according to the following pattern:

- New & Improved! **Chapter Opener:** Each chapter opens with an introductory activity that introduces some of the principal topics of the chapter by means of *cooperative learning*. They are followed by a "Key Ideas" feature that shows the interconnections among the various parts of mathematics previously discussed and between mathematics and the real world. Beyond, there are, within the body of the chapters, small groups to explore the concepts under discussion.
- **Examples** are often presented in a *problem-solving mode*, asking students to independently obtain a solution that can be compared with the solution presented in the text. Solutions are frequently structured in the Pólya four-step format.
- **Think Clouds.** These notes serve as quick reminders and clarify key points in discussions.
- **Cooperative Investigations:** Each chapter includes a number of games, puzzles, and explorations to be completed in small groups. Most of these can be adapted for future elementary and middle school classrooms.
- **Common Core State Standards—Standards for Mathematical Practice:** In addition to content, the Common Core advocates Standards for Mathematical Practice (SMP), which will help elementary and middle school students develop a deeper conceptual understanding of the math they are taught. Nearly all chapters have two "SMP" symbols in the margin noting a particular

standard along with highlighted text. This is to help you make connections between the standards and the content and eventual implementation.

- **From the NCTM Principles and Standards.** Extensive excerpts from the *NCTM Principles and Standards* help you understand the relevance of topics and what students will be expected to teach.
- **Problem Sets** are organized according to the following categories:
  - **Understanding Concepts** reinforce basic concepts and provide ample practice opportunities.
  - New & Improved! **Into the Classroom** problems pose questions that cause you to carefully consider how you might go about clarifying subtle and often misunderstood points for your future students. Answering these questions often forces one to think more deeply and come to a better understanding of the subtleties involved, especially in a student classroom. Group or cooperative problems are included in this section. The number of such problems has increased significantly in this edition.
  - New & Improved! **Responding to Students** exercises provide future teachers the opportunity to see what mathematical questions and procedures children will come up with on their own and ways to respond to them. Many more have been added to this edition including more from middle school.
  - **Thinking Critically** problems offer problem-solving practice related to the section topic. Many of these problems can be used as classroom activities or with small groups.
  - **Making Connections** problems apply the section concepts to solving real-life problems and to other parts of mathematics.
  - **State Assessment** exercises are problems and problem types from various state exams providing insight into the standardized testing based on state standards in effect prior to adoption of the Common Core standards. Common Core assessment is under development at this time.
  - **Writing** exercises are interspersed throughout the problem types providing opportunities to convey ideas through written words and not just numbers and symbols.
- **Chapter in Relation to Future Teachers** is a brief essay that discusses the importance of the material just covered in the context of future teaching and helps place the chapter in relation to the remainder of the book.
- **End-of-Chapter Material** Each chapter closes with the following features:
  - **Chapter Summary** is in a table format, with more complete information, to make it more helpful for reviewing the content. The summary includes *Key Concepts, Vocabulary, Definitions,* and *Notation,* and may also include *Theorems, Properties, Formulas, Procedures,* and *Strategy.*
  - **Chapter Review Exercises** help students self-check their understanding of the concepts discussed in the chapter.

Note for the Instructor

The principal goals of this text are to impart mathematical reasoning skills, a deep conceptual understanding, and a positive attitude to those who aspire to be elementary or middle school teachers. To help meet these goals, we have made a concerted effort to involve students in mathematical learning experiences that are intrinsically interesting, often surprising, aesthetically pleasing, and focused on **mathematical knowledge for teaching**. With enhanced skill at mathematical reasoning and a positive attitude toward mathematics come confidence and an increased willingness to learn the mathematical content, skills, and effective teaching techniques necessary to become a fine teacher of mathematics.

In our own classes, we have found it extremely worthwhile to spend considerable time on Chapter 1. Problem solving has gone a long way toward changing student attitudes and promoting their ability to reason mathematically. A course that begins and continues with an extensive study of the number systems and algorithms of arithmetic is not attractive or interesting to students who feel that they already know these things and have found them dull. By contrast, the material in Chapter 1 and the many problems in the problem sets are new, stimulating, and not what students have previously experienced. We have found that, aside from increasing interest, the extensive time spent on Chapter 1 develops positive attitudes, an increasing mathematical knowledge for teaching, and skills that make it possible to deal much more quickly with the usual material on number systems, algorithms, and all the subsequent ideas that are important to the teaching of mathematics in elementary schools.

There are a number of different ways to use the text. Some instructors prefer to intersperse topics from Chapter 1 throughout their courses as they cover subsequent chapters. Another approach is to begin with Chapters 2 and 3, and present Chapter 1, and then continue with additional chapters.

We have also found that it is important to answer the frequently asked question, "Why are we here?" by going beyond the discussions of conceptual understanding and showing the kinds of questions that children may ask. The Integrating Mathematics and Pedagogy (IMAP) videos in MyMathLab are an especially useful tool. In some of the videos, the children understand the material well and in others they are confused; both serve a valuable purpose. One hour spent early in the course with a few well-chosen video clips is a tremendous help in answering the "Why?" question. There are also assignable IMAP video homework problems in MyMathLab.

Prerequisite Mathematical Background

This text is for use in mathematics content courses for prospective elementary and middle school teachers. We assume that the students enrolled in these courses have completed two years of high school algebra and one year of high school geometry. We do not assume that the students will be highly proficient in algebra and geometry, but rather that they have a basic knowledge of those subjects and reasonable arithmetic skills. Typically, students bring widely varying backgrounds to these courses, and this text is written to accommodate that diversity.

Course Flexibility

The text contains ample material for either two or three semester-length courses at Washington State University, at which elementary education majors are required to take two three-semester hour courses, with the option for an elective third course that is particularly suited to the needs of upper elementary and middle school teachers. Our text is used in all three courses. The following suggestions are for single semester-length courses, but instructors should have little difficulty selecting material that fits the coverage needed for courses in a quarter system:

- A first course, *Problem Solving and Number Systems*, covers Chapters 1 through 7. Our own first course devotes at least five weeks to Chapter 1. The problem-solving skills and enthusiasm developed in this chapter make it possible to move through most of the topics in Chapters 2 through 7 more quickly than usual. However, as noted earlier, some instructors prefer to intersperse topics from Chapter 1 among topics covered later in their courses. There is considerable latitude in which topics an instructor might choose to give a lighter or heavier emphasis. One section, Section 3.5, isolates bases other than ten. This approach allows students to focus on various kinds of positional systems that are not decimal.

- A second course, *Algebra, Basic Geometry, Statistics, and Probability*, covers Chapters 8 through 14, with the optional inclusion of computer geometry software.
- An alternative approach, *Informal Geometry*, covers Chapters 9 through 12, with an instructor deciding on software if needed. Many instructors may want to have their students become familiar with dynamic geometry software such as GeoGebra, which is now available as a free download.
- Once the basic notions and symbolism of geometry have been covered in Sections 9.1 and 9.2, the remaining chapters in geometry can be taken up in any order. Section 9.3, on figures in space, should be covered before the instructor takes up surface area and volume in Sections 10.4 and 10.5
- Many universities use the text for a three-course sequence: "Problem Solving and Number Systems" (Chapters 1–7), "Algebra and Geometry" (Chapters 8–12), and "Probability and Statistics" (Chapters 13 and 14).

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Supplements

Student Supplements

Mathematics Activities for Elementary Teachers, Seventh Edition

Dan Dolan, Jim Williamson, and Mari Muri.

- Provides hands-on, manipulative-based activities keyed to the text that involve future elementary school teachers discovering concepts, solving problems, and exploring mathematical ideas.
- Colorful, perforated paper manipulatives in a convenient storage pack.
- Activities can be adapted for use with elementary students at a later time.

Video Resources

Video lectures in MyMathLab make it easy and convenient for students to watch the videos from anywhere and are ideal for distance learning or supplemental instruction. Videos include optional subtitles.

Instructor Supplements

Instructor's Solutions Manual

Beverly Fusfield

- Provides complete solutions to all problems in the text.

Instructor's Testing Manual

- Contains prepared tests with answer keys for each chapter.

Instructor's Guide to *Mathematics Activities for Elementary Teachers,* Seventh Edition

- Contains answers for all activities, as well as additional teaching suggestions for some activities.

PowerPoint Lecture Presentation

- Fully editable lecture slides include definitions, key concepts, and examples for every section of the text.

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MyMathLab from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy to use format.

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Specific to This MyMathLab Course:

- New! "Show Work" questions enable professors to assign questions that require more detailed solutions to prove conceptual understanding, which is highly emphasized in the new Common Core State Standards.
- Problems requiring students to watch Integrating Mathematics and Pedagogy (IMAP) videos test students' understanding of concepts and content in the context of children's reasoning.
- New! Flashcards reinforce key vocabulary from the text in an interactive online format.
- New! Getting Ready section provides an opportunity for remediation in areas where students may need a little more support.

About the Authors

Calvin Long received his B.S. in mathematics from the University of Idaho. Following M.S. and Ph.D. degrees in mathematics from the University of Oregon, he worked briefly as an analyst for the National Security Agency and then joined the faculty at Washington State University. His teaching ran the gamut from elementary algebra through graduate courses and frequently included teaching the content courses for prospective elementary school teachers.

His other professional activities include serving on numerous committees of the National Council of Teachers of Mathematics and the Mathematical Association of America, and holding various leadership positions in those organizations. Professor Long has also been heavily engaged in directing and instructing in-service workshops and institutes for teachers at all levels, has given more than 100 presentations at national and regional meetings of NCTM and its affiliated groups, and has presented invited lectures on mathematics education abroad.

Professor Long has coauthored two books and is the sole author of a text in number theory. In addition, he has authored over 90 articles on mathematics and mathematics education and also served as a frequent reviewer for a variety of mathematics journals, including *The Arithmetic Teacher* and *The Mathematics Teacher*. In 1986, he received the Faculty Excellence Award in Teaching from Washington State University, and in 1991, he received a Certificate for Meritorious Service to the Mathematical Association of America.

Aside from carrying out his professional activities, Cal enjoys listening to, singing, and directing classical music; reading; fly fishing; camping; and backpacking.

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Rich enjoys traveling, writing about mathematics, losing golf balls, listening to music, and going to plays and movies. He also loves and is enormously proud of his grandchildren, with whom he enjoys discussing the conceptual basis of mathematics, among other topics.

1 Thinking Critically

COOPERATIVE INVESTIGATIONS

The Gold Coin Game

Material Needed

15 markers (preferably circular and yellow, if possible) for each pair of students.

Directions

This is a two-person game. Each pair of players is given 15 gold coins (markers) on the desktop. Taking alternate turns, each player removes one, two, or three coins from the desktop. The player who takes the last coin wins the game. Play several games, with each player alternately playing first. Try to devise a winning strategy, first individually as you play and then thinking jointly about how either the first or second player can play so as to force a win.

Questions to Consider

1. To discover a winning strategy, it might be helpful to begin with fewer coins. Start with just 7 coins, and see if it is possible for one player or the other to play in such a way as to guarantee a win. Try this several times, and do not move on to question 2 until the answer is clear from what happened with 7 coins.
2. This time, start with 11 coins on the desktop. Is it now possible for one player or the other to force a win? Play several games until both you and your partner agree that there is a winning strategy, and then see how the player using that strategy should play.
3. Extend the strategy you developed in step 2 to the original set of 15 coins.
4. Would the strategy work if you began with 51 markers? Explain carefully and clearly.

Variation

Devise a similar game in which the player taking the last coin *loses* the game, and explain how one player or the other can force a win for your new game.

CHAPTER PREVIEW

This first chapter is dedicated to how one goes about solving a mathematical problem and how one learns to reason mathematically. Each problem to be solved needs some thought. In order to help the reader answer the questions, we present a large number of strategies for problem solving. The key question then is, "Which of the strategies should I use?" The answer is to do many problems for practice and you will ultimately instinctively go to the appropriate strategy for answering the problem. Of course, there may be many different ways to attack a problem, so it is important to try a number of strategies until you find one that works.

Why should a text devoted to future teachers focus on problem solving and mathematical reasoning? One of the most prominent features of current efforts to reform and revitalize mathematics instruction in American schools has been the recommendation that such instruction should stress problem solving and quantitative reasoning. That this emphasis continues is borne out by the fact that it appears as the first of the process standards in the National Council of Teachers of Mathematics' (NCTM's) *Principles and Standards for School Mathematics*, published in 2000. (See the Problem-Solving Standard on the next page and page 14 of the preface.) Children need to learn to *think* about quantitative situations in insightful and imaginative ways—just memorizing seemingly arbitrary rules for computation is unproductive.

Of course, if children are to learn problem solving, their teachers must themselves be good teachers of problem solving. Thus, the purpose of this chapter, and indeed of this entire book, is to help you to think more critically, analytically, and thoughtfully, in order to be more comfortable with mathematical reasoning and discourse and to bring those mathematical habits of the mind to your classroom.

These traits are a part of the Common Core State Standards for Mathematics, which we will call **"Common Core"** throughout and will be described more completely at the end of the next section.

KEY IDEAS

- The need to look for patterns: using inductive reasoning to form a conjecture
- Beginning to understand the Standards for Mathematical Practice of the Common Core State Standards for Mathematics (also known as CCSS-M) and the *Principles and Standards for School Mathematics* of the NCTM
- The four problem-solving principles of George Pólya
- A variety of available problem-solving strategies (12 are highlighted in this chapter, with more to come later)
- The idea of algebra as a problem-solving strategy
- The use of the Pigeonhole Principle
- Deductive reasoning
- The rule of indirect reasoning

FROM THE NCTM PRINCIPLES AND STANDARDS

Problem-Solving Standard

Instructional programs from prekindergarten through grade 12 should enable all students to—

- *build new mathematical knowledge through problem solving;*
- *solve problems that arise in mathematics and in other contexts;*
- *apply and adapt a variety of appropriate strategies to solve problems;*
- *monitor and reflect on the process of mathematical problem solving.*

Problem solving is the cornerstone of school mathematics. Without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills are severely limited. Students who can efficiently and accurately multiply but who cannot identify situations that call for multiplication are not well prepared. Students who can both develop *and* carry out a plan to solve a mathematical problem are exhibiting knowledge that is much deeper and more useful than simply carrying out a computation. Unless students can solve problems, the facts, concepts, and procedures they know are of little use. The goal of school mathematics should be for all students to become increasingly able and willing to engage with and solve problems.

Problem solving is also important because it can serve as a vehicle for learning new mathematical ideas and skills (Schroeder and Lester 1989). A problem-centered approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships.

1.1 An Introduction to Problem Solving

The problem that follows is an excellent and realistic example of problem solving that works well with fifth-grade students. Look for how many different ways there are to solve this problem and how many mathematical discussions there can be in a classroom.

When the children arrived in Frank Capek's fifth-grade class one day, this "special" problem was on the blackboard:

> *Old MacDonald had a total of 37 chickens and pigs on his farm. All together, they had 98 feet. How many chickens were there and how many pigs?*

After organizing the children into problem-solving teams, Mr. Capek asked them to solve the problem. "Special" problems were always fun and the children got right to work. Let's listen in on the group with Mary, Joe, Carlos, and Sue:

"I'll bet there were 20 chickens and 17 pigs," said Mary.

"Let's see," said Joe. "If you're right there are 2×20*, or 40, chicken feet and* 4×17*, or 68, pig feet. This gives 108 feet. That's too many feet."*

"Let's try 30 chickens and 7 pigs," said Sue. "That should give us fewer feet."

"Hey," said Carlos. "With Mary's guess we got 108 feet, and Sue's guess gives us 88 feet. Since 108 is 10 too much and 88 is 10 too few, I'll bet we should guess 25 chickens—just halfway between Mary's and Sue's guesses!"

These children are using a **guess and check** strategy. If their guess gives an answer that is too large or too small, they adjust the guess to get a smaller or larger answer as needed. This can be a very effective strategy. By the way, is Carlos's guess right?

Let's look in on another group:

"Let's make a table," said Nandita. "We've had good luck that way before."

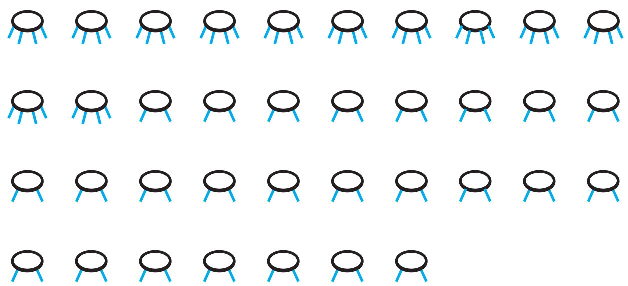
"Right, Nani," responded Ann. "Let's see. If we start with 20 chickens and 17 pigs, we have 2×20*, or 40, chicken feet and* 4×17*, or 68, pig feet. If we have 21 chickens, ..."*

This is a powerful refinement of guess and check.

Chickens	Pigs	Chicken Feet	Pig Feet	Total
20	17	40	68	108
21	16	42	64	106
22	15	44	60	104
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Making a table to look for a pattern is often an excellent strategy. Do you think that the group with Nandita and Ann will soon find a solution? How many more rows of the table will they have to fill in? Can you think of a shortcut?

Mike said, "Let's draw a picture. We can draw 37 circles for heads and put two lines under each circle to represent feet. Then we can add two extra feet under enough circles to make 98. That should do it."



Drawing a picture is often a good strategy. Does it work in this case?

"Oh! The problem is easy," said Jennifer. "If we have all the pigs stand on their hind legs, then there are 2×37*, or 74, feet touching the ground. That means that the pigs must be holding 24 front feet up in the air. This means that there must be 12 pigs and 25 chickens!"*

$$\begin{array}{r} 98 \\ -74 \\ \hline 24 \end{array} \qquad \begin{array}{r} 37 \\ -12 \\ \hline 25 \end{array}$$

It helps if you can be ingenious like Jennifer, but it is not essential, and children *can* be taught strategies like the following:

Guess and Check

Make a Table

Look for a Pattern

Draw a Picture